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**Total Marks: 04**

**Marks Obtained:**

**Numerical Computing**

**Assignment #03**

**Submitted To: Sir Sajjad Ghori**

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**Question no.1**

Prove the formulas for first and second-order derivatives using **Newton’s Forward Difference Formula**:

F ́ (a) = [ Δ f(a) - Δ2 f(a) + ⅓Δ3 f(a) - Δ4 f(a) + . . .]

F ́ ́ (a) = [ Δ2 f(a) - Δ3 f(a) + Δ4 f(a) - Δ5 f(a) + - . . .]

**Solution**

Let

Y= f(x) ⸫ assume we have to find the derivative of y

**Newton’s forward difference formula:**

F(x) = f(x0) + s Δ f(x0) + Δ2 f(x0) + Δ3 f(x0) + Δ4 f(x0) + …

Now we simplify the formula:

F(x) = f(x0) + s Δ f(x0) + Δ2 f(x0) + Δ3 f(x0) + Δ4 f(x0) + … (eq 1)

As we know that

**S =**  (eq 2)

Differentiation (eq 1) W.R.T s

Δ f(x0) + Δ2 f(x0) + Δ3 f(x0) + Δ4 f(x0) + …

Differentiation (eq 2) W.R.T x

Now

[ Δ f(x0) + Δ2 f(x0) + Δ3 f(x0) + Δ4 f(x0) + . . .] (eq 3)

At x = x0,

Hence Putting s = 0 in (eq 3)

**First derivative:**

= [ Δ f(a) - Δ2 f(a) + ⅓Δ3 f(a) - Δ4 f(a) + . . .]

Again, differentiation on (eq 3) we have

=

[ Δ f(x0) + Δ2 f(x0) + Δ3 f(x0) + Δ4 f(x0) + . . .]

[ Δ2 f(x0) + Δ3 f(x0) + Δ4 f(x0) + . . .] (eq 4)

At x = x0,

Hence Putting s = 0 in (eq 4)

**Second derivative:**

= [ Δ2 f(a) - Δ3 f(a) + Δ4 f(a) - Δ5 f(a) + - . . .]

**Hence Proved!**

**Question no.2**

Prove the formulas for first and second-order derivatives using **Newton’s Backward Difference Formula**:

F ́ (a) = [ Δ f(a) + Δ2 f(a) + ⅓Δ3 f(a) + Δ4 f(a) + - . . .]

F ́ ́ (a) = [ Δ2 f(a) + Δ3 f(a) + Δ4 f(a) + Δ5 f(a) + - . . .]

**Solution**

Let

Y= f(x) ⸫ assume we have to find the derivative of y

**Newton’s backward difference formula:**

F(x) = f(x0) + s Δ f(x0) + Δ2 f(x0) + Δ3 f(x0) + Δ4 f(x0) + …

Now we simplify the formula:

F(x) = f(x0) + s Δ f(x0) + Δ2 f(x0) + Δ3 f(x0) + Δ4 f(x0) + … (eq 1)

As we know that

**S =** (eq 2)

Differentiation (eq 1) W.R.T s

Δ f(x0) + Δ2 f(x0) + Δ3 f(x0) + Δ4 f(x0) + …

Differentiation (eq 2) W.R.T x

Now

[ Δ f(x0) + Δ2 f(x0) + Δ3 f(x0) + Δ4 f(x0) + . . .] (eq 3)

At x = x0,

Hence Putting s = 0 in (eq 3)

**First derivative:**

= [ Δ f(a) + Δ2 f(a) + ⅓Δ3 f(a) + Δ4 f(a) + . . .]

Again, differentiation on (eq 3) we have

=

[ Δ f(x0) + Δ2 f(x0) + Δ3 f(x0) + Δ4 f(x0) + . . .]

[ Δ2 f(x0) + Δ3 f(x0) + Δ4 f(x0) + . . .] (eq 4)

At x = x0,

Hence Putting s = 0 in (eq 4)

**Second derivative:**

= [ Δ2 f(a) + Δ3 f(a) + Δ4 f(a) + Δ5 f(a) + . . .]

**Hence Proved!**

**Question no.3**

Prove the Numerical Integration formula for the following rules:

1. **Trapezoidal Rule**

**Solution**

We know that quadrative formula is

Putting n = 1 and neglecting second and higher other differences we get

Similarly,

P.T.O

*.*

*.*

*.*

Adding the results

But Property of differential integral

Replace ‘y’ with ‘f’

**Hence Proved!**

1. **Simpsons’ 1/3rd Rule**

**Solution**

We know that quadrative formula is

Putting n = 2 and neglecting third and higher other differences, we get

Similarly,

P.T.O

*.*

*.*

*.*

Adding the results (on adding the above Equations)

Replace ‘y’ with ‘f’

**Hence Proved!**

**Question no.4**

Find the first order derivative accurately up to 8th order using Richardson Extrapolation at x = 5 employing step sizes of h1 = 0.5, h2 = 0.25, h3 = 0.125, h4 = 0.0625.

F(x) = -0.1x4 - 0.15x3 - 0.5x2 -0.25x +1.2

**Solution**

**h D(h) D1(h) D2(h) D3(h)**

h 0.5 -1

-0.9125

0.25 -0.9344 -0.9120

-0.912 0.9123

0.125 -0.9176 -0.9123

-0.9123

0.0625 -0.9136

D(h) = [ f (xi + h) - f (xi - h)]

D (0.5) = [ f (0.5 + 0.5) - f (0.5 - 0.5)] => 1[f (1) – f (0)]

= 1 [0.2 – 1.2] **D (0.5) = -1**

D (0.25) = [ f (0.5 + 0.25) - f (0.5 - 0.25)] => 2[f (0.75) – f (0.25)]

= 2[0.6363 – 1.1035] **D (0.25) = -0.9344**

D (0.125) = [ f (0.5 + 0.125) - f (0.5 - 0.125)] => 4[f (0.625) – f (0.375)]

= 4[0.7966 – 1.0260] **D (0.125) = -0.9344**

D (0.0625) = [ f (0.5 + 0.0625) - f (0.5 - 0.0625)] => 8[f (0.5625) – f (0.4375)]

= 8[0.8645 – 0.9787] **D (0.0625) = -0.9136**

D1(h) = = = **-0.9125**

D1= = = **-0.912**

D1 = = = **-0.9123**

D2(h) = = = **-0.9120**

D2= = = **-0.9123**

D3(h) = = = **-0.9123**